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MICROLENSING RATES FROM SELF-CONSISTENT GALACTIC MODELS

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Abstract

We study different models of dark matter distribution for the halo of our galaxy. In particular, we consider Eddington and King-Michie models, which include anisotropy in the velocity space, and compute in a self-consistent way the amount of dark matter present in the halo. Assuming that the dark matter is in form of Massive Astrophysical Compact Halo Objects (MACHOs), we find for each model the expected number of microlensing events and their average time duration for an experiment monitoring stars in the Large Magellanic Cloud (LMC). The main effect of including anisotropy is to reduce the microlensing rate by about 30% and to increase, but only slightly, the mean event duration, as compared to the standard halo model. Consideration of different luminous models for the visible part of the galaxy also induce variations in the microlensing results by roughly the same amount as mentioned above. The main uncertainty, in order to be able to discriminate between different dark matter distributions and to estimate the fraction of it in form of MACHOs, is due to the poor knowledge of the rotation velocity at large galactocentric distances.

1. INTRODUCTION

An important problem in astrophysics lies in the knowledge of the nature of the non-luminous matter present in galactic halos as well as of their shape and extension. The dark matter in galactic halos may be in the form of MACHOs (Massive Astrophysical Compact Halo Objects), in the mass range $10^{-7} < M/M_\odot <\sim 10^{-1}$ (De Rújula et al. 1992) , which can be detected using the gravitational lensing effect (Paczyński 1986). The few microlensing events found so far by monitoring stars in the Large Magellanic Cloud (LMC) by the EROS (Aubourg et al. 1993) and the MACHO (Alcock et al. 1993) collaborations, while confirming the presence of MACHOs, do not yet allow to make a precise estimate of the fraction of the halo dark matter in the form of MACHOs nor to infer whether they are located in the halo or rather either in the LMC itself (Sahu 1994, Wu 1995, Gould et al. 1994) or in a thick disk of our galaxy. Assuming a standard spherical halo model, Alcock et al. (1995a) found that MACHOs contribute a fraction $0.19^{+0.16}_{-0.10}$ to the halo dark matter, whereas their average mass turns out to be $\sim 0.08 M_\odot$ (Jetzer 1994).

In this paper we investigate different models of dark matter distribution for the halo of our galaxy. In particular we consider the so-called Eddington and King-Michie models, which include anisotropy effects in the velocity space. For these models we determine in a self-consistent way the amount of dark matter present in the halo. Moreover, assuming that the dark matter is in form of MACHOs we find for each model the expected number of microlensing events and their average time duration for an experiment monitoring stars in the LMC. These results are of relevance in order to determine the fraction of the halo dark mass in form of MACHOs. If the preliminary results of Alcock et al. (1995a) mentioned above will be confirmed by future improved observations, the problem arises of how to explain the nature of the remaining fraction of the halo dark matter. Besides the possibility of being composed of new exotic particles, it could still be baryonic and in form of molecular clouds (mainly H_2). This latter scenario has been recently investigated in detail by us in several papers (De Paolis et al. 1995, 1995a) and also by Gerhard and Silk (1995).

In section 2 we describe the models for the dark matter distribution and in section 3 we give the formulas for computing the microlensing rate, the average time duration of an event and the average mass of the MACHOs using the moment method. The numerical results and the conclusions are given in section 4 and 5, respectively.

2. MODELS FOR DARK MATTER DISTRIBUTION

We assume that the galaxy contains two main components, namely, the visible component and the dark component. In the following we will assume that the dark matter consists of MACHOs, although the results on the total amount of the dynamical mass do not depend on this assumption and are,

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therefore, of more general validity. In the last few years, the picture of the visible component of the Milky Way has evolved from the BSS model (Bahcall et al. l983) which assumed stars to be distributed according to a central bulge, a spheroid and an exponential disk. In particular, the central concentration of stars is now described by a triaxial bulge model with the density law

$$\rho_C(x,y,z) = \frac{M_b}{8\pi\tilde{a}bc}e^{-s^2/2} \ , \quad \text{with} \quad s^4 = (x^2/\tilde{a}^2 + y^2/b^2)^2 + z^4/c^4 \ , \qquad (1)$$

where the bulge mass is $M_b \sim 2 \times 10^{10} \ M_\odot$ and the scale lengths are $\tilde{a} = 1.49$ kpc, $b = 0.58$ kpc, $c = 0.40$ kpc (Dwek et al. 1994). The coordinates x and y span the galactic disk plane, whereas z is perpendicular to it. The remaining luminous matter (the spheroid and the disk of the BSS model) can be described with a double exponential disk (see Gilmore et al. l989), so that the galactic disk has both a "thin" (D_1) and a "thick" (D_2) component. For the "thin" luminous disk we adopt the following density distribution

$$\rho_D(X,z) = \frac{\Sigma_0}{2H} \ e^{-|z|/H} \ e^{-(X-R_0)/h}, \qquad (2)$$

where the local projected mass density is $\Sigma_0 \sim 25 \ M_\odot \ \text{pc}^{-2}$, the scale parameters are $H \sim 0.30$ kpc and $h \sim 3.5$ kpc and $R_0{=}8.5$ kpc is the local galactocentric distance. Here X is the galactocentric distance in the plane. For the "thick" component we consider the same density law as in eq.(2) but with variable thicknesses in the range $H = 1 \pm 0.5$ kpc and local projected density $\Sigma_0 \sim 50 \pm 25 \ M_\odot \ \text{pc}^{-2}$.

The total local projected mass density within a distance of $(0.3-1.1)$ kpc of the galactic plane is measured to be in the range $(40-85) \ M_\odot \ \text{pc}^{-2}$. This explains our chosen range of values for the "thick" disk, which in fact corresponds to a total luminous projected mass density of $(50-100) \ M_\odot \ \text{pc}^{-2}$. In our models we also consider the effect of varying the bulge mass $M_b = 2 \pm 1 \times 10^{10} \ M_\odot$, the local galactocentric distance $R_0 = 8.5 \pm 1$ kpc and the scale length $h \sim 3.5 \pm 0.5$ kpc, while we keep all the remaining parameters appearing in the previous equations fixed.

We treat MACHOs composing the dark matter with the formalism based on the equation of state assuming that they are spherically symmetric distributed and that:

i) the velocities of MACHOs at any point of the galaxy are limited by the escape velocity $v_c^2(r)$ from the galaxy itself;

ii) the distribution function can be anisotropic in the velocity space as a consequence of the initial conditions of the galaxy formation as well as of the MACHO formation processes [1]. Then, for MACHOs of the same mass M, we adopt the

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[1]It is well known that for elliptical galaxies models in which the luminous matter is described by distribution functions with anisotropy in velocity space can account for their shapes. Moreover both isotropic and anisotropic models can explain optical and X-ray observations (De Paolis et al. 1995b).

King-Michie distribution function (King 1966, Michie 1963), which can be written as (Ingrosso et al. 1992)

$$dn(r) = A(2\pi\sigma^2)^{-3/2} \; e^{[W(r)-W(0)]} \; (e^{-v^2/2\sigma^2} - e^{-W(r)}) \; e^{-L^2(r)/2L_c^2} \; d^3v \; , \quad (3)$$

for $v \leq v_c(r)$ and $dn = 0$ otherwise. Here $W(r) = v_c^2(r)/2\sigma^2$ is the energy cutoff parameter, $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ the angular momentum and L_c the angular momentum cutoff. Moreover A is a normalization constant and σ a parameter which in the limit of the classical statistics ($W(r) \rightarrow \infty$ and $L_c \rightarrow \infty$) represents the one-dimensional MACHO velocity dispersion. It is clear that the above distribution function gives lower values for the phase space density at large values of energy and angular momentum as compared to the ones obtained from the Boltzmann distribution. Furthermore it introduces an anisotropy in the velocity space, which increases with the radial coordinate r and leads to highly eccentric orbits for the MACHOs located in the outer regions of the galaxy.

Visible and dark components are considered to be in hydrostatic equilibrium in the overall gravitational potential V solution of the Poisson equation $\nabla^2 V = -4\pi G(\rho_H + \rho_C + \rho_{D_1} + \rho_{D_2})$, which we solve assuming, as stated, spherical symmetry for the dark mass distribution. Clearly, this assumption as well as the purely radial anisotropy in the phase space for MACHOs are first order approximations, since the presence of the "thin" and "thick" stellar disks distort both the density and the velocity distribution of MACHOs close to the disk. However, we are interested mainly in the microlensing rate expected looking towards the LMC and we have verified that the errors in the number of events due to MACHOs close to the disks (i.e. in the region where the errors due to our approximations are higher) are negligible respect to those due to the uncertainties in the determination of the visible component.

The distribution function given in eq.(3) is the most natural way to build self-consistent galactic models, since the usual Boltzmann statistics requires an external cutoff (in the density) to avoid the mass divergence. Vice versa eq.(3) naturally implies a MACHO mass density equal to zero at the boundary R of the galaxy where $W(R) = 0$. Starting from eq.(3) and expressing $L_c = Mr_a\sigma$ in terms of the anisotropy radius r_a (Binney & Tremaine 1987), we obtain, after an integration over d^3v, the radial component for the mass density

$$\rho_H(r) = A \; (2\pi\sigma^2)^{-3/2} \; e^{[W(r)-W(0)]} \; \left(\frac{r_a}{r}\right) \; \int_0^{W(r)} [e^{-\xi} - e^{-W(r)}] \; F(\lambda)d\xi \; , \quad (4)$$

where $\lambda = (r/r_a)\sqrt{\xi}$, $\xi = v^2/2\sigma^2$, $F(\lambda)$ is the Dawson integral (Abramowitz & Stegun 1965) and $W(r) = -V(r)/\sigma^2$.

The distribution function in eq.(3) can be approximated in the limit without energy cutoff ($W \rightarrow \infty$) with a well treatable analytical model. The so-called Eddington model (Binney & Tremaine l987) for which

$$dn(r) = A(2\pi\sigma^2)^{-3/2} \; e^{[W(r)-W(0)]} \; e^{-v^2/2\sigma^2} \; e^{-L^2(r)/2L_c^2} \; d^3v \; . \quad (5)$$

In this case one can easily show that for values of r greater than the halo core radius a (the region in which the galactic dynamical mass is dominated by the halo dark matter), the dark mass density $\rho_H(r)$ can be approximated by (Binney & Tremaine 1987)

$$\rho_H(r) = \rho_0 \left(\frac{a^2 + R_0^2}{a^2 + r^2} \right) \frac{1}{1 + (r/r_a)^2} \ , \tag{6}$$

where ρ_0 is the local dark mass density.

Before going on with the presentation of the numerical results for the dark matter distributions, we discuss how we compute the microlensing rate and the average time duration.

3. MICROLENSING RATES AND MASS MOMENTS

When a MACHO of mass M is sufficiently close to the line of sight between us and a star in the LMC, the light from the source suffers a gravitational deflection and the original star brightness increases by

$$A = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}} \ . \tag{7}$$

Here $u = d/R_E$ (d is the distance of the MACHO from the line of sight) and R_E is the Einstein radius defined as:

$$R_E^2 = \frac{4GMD}{c^2} \tilde{x}(1 - \tilde{x}) \ , \tag{8}$$

with $\tilde{x} = s/D$, where D and s are the distance between the source (a star in the LMC), respectively the MACHO and the observer.

Microlensing rates depend on the mass and velocity distribution of MACHOs. The mass density at a distance s from the observer is given by eq.(4) or (6). However, the adopted halo model does not determine the MACHO number density as a function of mass. A simplifying assumption is to let the mass distribution be independent of the position in the galactic halo, i.e., we assume the following factorized form for the number density per unit mass dn/dM,

$$\frac{dn}{dM} dM = \frac{dn_0}{d\mu} \frac{\rho_H(\tilde{x})}{\rho_0} d\mu = \frac{dn_0}{d\mu} H(\tilde{x}) d\mu \ , \tag{9}$$

with $\mu = M/M_\odot$, $n_0(\mu)$ not depending on $\tilde{x}$ and is subject to the normalization $\int d\mu \frac{dn_0}{d\mu} M = \rho_0$. Nothing a priori is known about the distribution $dn_0/d\mu$.

A different situation arises for the MACHOs velocity distribution, whose projection in the plane perpendicular to the line of sight $f(\tilde{x}, v_T)$ is obtained from eq.(3), by adopting cylindrical coordinates along the microlensing tube (so that $d^3v = v_T dv_T dv_x d\theta$). After integration over the longitudinal velocity v_x

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and the direction θ of the transverse velocity v_T in the perpendicular plane we get

$$f(\tilde{x}, v_T) = (2\pi\sigma^2)^{-3/2} \ A \ e^{[W(r)-W(0)]} \int_{-v_c(r)}^{+v_c(r)} \ dv_x \ [e^{-(v_x^2+v_T^2)/2\sigma^2} - e^{-W(r)}]$$

$$\int_0^{2\pi} \ d\theta \ e^{-(r/r_a)^2[(1-sin^2\gamma cos^2\theta)v_T^2/2\sigma^2+sin^2\gamma v_x^2/2\sigma^2-sin2\gamma cos\theta v_x v_T/2\sigma^2]} \ ,$$
$$(10)$$

where γ is the angle between the MACHO radial distance $\mathbf{r}$ and the distance $\mathbf{s}$ from the observer. The average value $< v_T^m >$ is obtained by a further integration

$$< v_T^m(\tilde{x}) > = \int_0^{+v_c(r)} f(\tilde{x}, v_T) v_T^{(m+1)} dv_T \ . \tag{11}$$

In the case of anisotropic models with $W \to \infty$ (the Eddington model in eq.(5)) the previous equation can be cast into the following form:

$$< v_T^m(\tilde{x}) > = \frac{\sqrt{2}\sigma^m}{[1+(r/r_a)^2 \ sin^2\gamma]^{1/2}} \ \int_0^\infty I_0[\eta^2 g(\tilde{x})] \ e^{-\eta^2[1+(r/r_a)^2+g(\tilde{x})]} \ \eta^{(m+1)} d\eta \ , \tag{12}$$

where I_0 is a Bessel function and $g(\tilde{x})$ is given by

$$g(\tilde{x}) = \frac{(r/r_a)^2[1+(r/r_a)^2] \ sin^2\gamma}{[1+(r/r_a)^2 \ sin^2\gamma]} \ . \tag{13}$$

In order to find the rate at which a single star is microlensed with amplification $A \geq A_{min}$ (or $u \leq u_{max}$), we consider MACHOs with masses between μ and $\mu + d\mu$, located at a distance from the observer in the range $\tilde{x}$ and $\tilde{x} + d\tilde{x}$ and with transverse velocity in the interval v_T and $v_T + dv_T$. The collision time can be calculated using the well-known fact that the inverse of the collision time is the product of the MACHO number density, the microlensing cross-section and the average velocity. The rate $d\Gamma$ at which a single star is microlensed in the interval $d\mu d\tilde{x} dv_T$ is given by (De Rújula et al. 1991, Griest 1991)

$$d\Gamma(\tilde{x}, \mu, v_T) = 2Dr_E u_{max} v_T^2 f(\tilde{x}, v_T)[\mu\tilde{x}(1-\tilde{x})]^{1/2} H(\tilde{x}) \frac{dn_0}{d\mu} d\mu d\tilde{x} dv_T, \tag{14}$$

with

$$r_E = \left(\frac{4GM_\odot D}{c^2}\right)^{1/2} \sim 3.2 \times 10^9 \ km \ . \tag{15}$$

One has to integrate the differential number of microlensing events, $dN_{ev} = N_\star t_{obs} d\Gamma$, over an appropriate range for μ, v_T and $\tilde{x}$, in order to obtain the total number of microlensing events which can be compared with an experiment monitoring $N_\star$ stars during an observation time t_{obs} and which is able to detect an amplification such that $A \geq A_{min}$.

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The range of integration for v_T is $0 < v_T < v_c(\tilde{x})$ for the King-Michie models, while the upper limit goes to ∞ for the Eddington models. Moreover, $\tilde{x}$ ranges between 0 and 1 and μ varies in the interval where the mass distribution $dn_0/d\mu$ is not vanishing.

However, each experiment has time thresholds T_{min} and T_{max} and only detects events with $T_{min} \leq T \leq T_{max}$, and thus the integration range has to be such that T lies in this interval. The total number of micro-lensing events is then given by

$$N_{ev} = \int dN_{ev} \ \Theta(T - T_{min})\Theta(T_{max} - T), \tag{16}$$

where the integration is over the full range of $d\mu d\tilde{x} dv_T$ mentioned above. T is related in a complicated way to the integration variables, because of this no direct analytical integration in eq.(16) can be performed.

In practice in order to evaluate eq.(16) we define an efficiency function $\epsilon_0(\mu)$ which measures the fraction of the total number of microlensing events that meet the condition on T at a fixed MACHO mass and thus takes into account the Θ functions in eq.(16). A more detailed analysis (De Rújula et al. 1991) shows that $\epsilon_0(\mu)$ is, to a very good approximation, equal to unity for possible MACHO objects in the mass range of interest. We will therefore set $\epsilon_0(\mu) = 1$ in the following.

From the experimental lensing data it is possible to extract information on the MACHO mass distribution (De Rújula et al. 1991). It proves most useful to average powers of the duration T, i.e., to construct "duration" moments. Consider an experiment that observes $N_\star$ stars during a time t_{obs}, that has recorded a set of microlensing events, each one with a duration T. For each event we get a value for the dimensionless variable τ, defined as follows:

$$\tau = \frac{v_0}{r_E} T = \frac{v_0}{v_T}[\mu \tilde{x}(1 - \tilde{x})]^{1/2} \ , \tag{17}$$

(here v_0 is the local rotational velocity) and we construct the n-moment of τ from experimental data as: $< \tau^n > = \sum_{events} \tau^n$.

The theoretical expectations for these moments are calculated as follows:

$$< \tau^n > = \int dN_{ev} \ \tau^n = V \ u_{max} \ \gamma(m) <\mu^m> \ , \tag{18}$$

with $m \equiv (n + 1)/2$ and

$$V \equiv 2N_\star t_{obs} D \, r_E \, v_0 \ , \tag{19}$$

$$\gamma(m) \equiv \int_0^{+v_c} \int_0^1 d\tilde{x} \left(\frac{v_T}{v_0}\right)^{1-n} v_T f(\tilde{x}, v_T)[\tilde{x}(1 - \tilde{x})]^m H(\tilde{x}) \ , \tag{20}$$

$$<\mu^m> \equiv \int d\mu \ \frac{dn_0}{d\mu}\mu^m. \tag{21}$$

The main point of eq.(18) is that the experimental value of $< \tau^n >$ allows to determine the moments of the MACHO mass distribution, which can be used to reconstruct the mass function itself. The mean local density of MACHOs (number per cubic parsec) is related to $< \mu^0 >$. The average local mass density of MACHOs is proportional to $< \mu^1 >$ (solar masses per cubic parsec). The mean MACHO mass $\bar{M}$ is then given by

$$\bar{M} \equiv \frac{< \mu^1 >}{< \mu^0 >} = \frac{< \tau^1 >}{< \tau^{-1} >} \frac{\gamma(0)}{\gamma(1)} \ . \tag{22}$$

where $< \tau^1 >$ and $< \tau^{-1} >$ are determined through the observed microlensing events. We may use $< \mu^1 >$ to compute the fraction f of the dark matter density ρ_0 that has been detected in the form of MACHOs. Indeed, we have

$$f = \frac{M_\odot}{\rho_0} < \mu^1 > \approx 126 < \mu^1 > \ \mathrm{pc}^3 \ \left(\frac{7.9 \times 10^{-3} \ M_\odot \ \mathrm{pc}^{-3}}{\rho_0} \right) \ . \tag{23}$$

Finally, the event duration T can be expressed in terms of the γ functions defined in eq.(20) as follows:

$$T = \frac{r_E}{v_0} < \mu^1 >^{1/2} \frac{\gamma(1)}{\gamma(1/2)} \ . \tag{24}$$

In order to quantify the expected number of events it is convenient to take as an example a delta function distribution for the mass, i.e. $dn_0/d\mu = \delta(\mu - \bar{\mu})/\mu$. The rate of microlensing events with $A \geq A_{min}$ (or $u \leq u_{max}$), is then

$$\Gamma(A_{min}) = \tilde{\Gamma} u_{max} = 2Dr_E u_{max} \frac{\rho_0}{M_\odot} \frac{1}{\sqrt{\mu}} \int_0^{+v_c} dv_T \int_0^1 d\tilde{x} \ v_T^2 f(\tilde{x}, v_T)[\tilde{x}(1-\tilde{x})]^{1/2} H(\tilde{x}) \ . \tag{25}$$

Assuming the standard halo model for the MACHO density distribution (which one obtains by taking the limit $r_a \to \infty$ in eq.(6)) $\rho_H(r) = \rho_0(a^2 + R_0^2)/(r^2 + a^2)$ with $\rho_0 = 7.9 \times 10^{-3} \ M_\odot \ \mathrm{pc}^{-3}$, $a = 5.5$ kpc, using for the LMC distance $D = 50$ kpc and for the angle between the line of sight and the direction of the centre of the galaxy $\alpha = 82^0$, we obtain (De Rújula et al. 1991, Jetzer 1991).

$$\tilde{\Gamma} = 4 \times 10^{-13} \frac{1}{s} \left(\frac{v_0}{215 \ \mathrm{km/s}} \right) \left(\frac{1}{\sqrt{D/\mathrm{kpc}}} \right) \left(\frac{\rho_0}{7.9 \times 10^{-3} \ M_\odot \ \mathrm{pc}^{-3}} \right) \frac{1}{\sqrt{M/M_\odot}} \ . \tag{26}$$

For an experiment monitoring $N_\star$ stars during an observation time t_{obs} the total number of events with a amplification $A \geq A_{min}$ is: $N_{ev}(A_{min}) = N_\star t_{obs} \Gamma(A_{min})$. In Table 1 we show some values of N_{ev} for the LMC, taking $t_{obs} = 1$ year, $N_\star = 10^6$ stars and $A_{min} = 1.34$.

4. RESULTS

We first discuss the Eddington model whose results coincide, in the isotropic limit, with those given in Table 1 for the galactic standard halo with flat rotation curve up to the LMC. We get a set of Eddington models by varying the values of the parameters a, ρ_0 and r_a. The amount of luminous matter, as discussed in section 2, is in the range between 4.8×10^{10} $M_\odot$ ("minimum disk model") and 1.1×10^{11} $M_\odot$ ("maximum disk model").

For each set of parameters we properly compute the resulting rotation curve considering the circular speed of the exponential disks (Binney & Tremaine 1987) and the contribution due to the bulge and dark halo. A given model is physically acceptable only if it leads to a flat rotation curve up to the distance of the LMC (with a local rotational velocity $v_0 = 215 \pm 10$ km s^{-1}). In order to check this requirement we perform a χ^2_{LMC} test, which gives a measure of the flatness of the rotation curve in the region 5 kpc $< r <$ 50 kpc. This way we find a range for the parameters for which the corresponding models satisfy the $\chi^2_{LMC} < 1$ test.

In Table 2 we report the mean values of the allowed range for the parameters a, ρ_0 and r_a. The errors quoted in the Table are such that $\sim 80\%$ of the allowed models, which fulfill the $\chi^2_{LMC} < 1$ test, have parameters whose values lie in the range defined by the errors around the mean quantity. Table 2 gives also the number of microlensing events N_{ev} (as in Table 1 we consider an experiment monitoring 10^6 stars during 1 year and for the MACHO we assume a mass of $10^{-1} M_\odot$), the average event duration T, the ratio $[\gamma(0)/\gamma(1)]$, which is related to the mean MACHO mass $\bar{M}$, as well as the amounts of dark mass M_H^{LMC} and M_H^{Tot} which are inside the distance to the LMC and 250 kpc, respectively. The latter value is of course arbitrary and should be regarded as an illustration, given also the fact that the true extent of the halo is unknown.

As can be seen from Table 2, the main effect of the anisotropy in velocity space (second line) is to reduce the amount of M_H^{Tot} substantially. The dark mass M_H^{LMC} also decreases with increasing anisotropy, however by less than 20%. This is due to the fact that the anisotropy in the velocity space becomes particularly important starting from a distance $r > r_a$ (see eq.(6)).

The decrease of M_H^{LMC} for anisotropic models compared to isotropic ones implies a smaller number of microlensing events N_{ev}, since the MACHOs orbits are now more eccentric. This reduction is also partially due to a shortening of the average transverse MACHO velocity $< v_T >$ with increasing anisotropy which, on the other hand, leads to an increase of the event duration. Finally, as one can see from the last column of Table 2, for the anisotropic models there is a decrease of the ratio $[\gamma(0)/\gamma(1)]$ leading (by eq.(22)) to a smaller average MACHO mass $\bar{M}$.

Till now we assumed a flat rotation curve in the range $(5-50)$ kpc. Actually, the galactic rotation curve is well measured only in the range $(5-20)$ kpc (Merrifield 1992). Thus we consider Eddington models for which we relax the condition of a flat rotation curve in the range $(20-50)$ kpc. To select acceptable physical models we follow the strategy outlined, e.g., in Gates et al. (1995). We

require that: i) the local rotational velocity is $v_0 = 215 \pm 10$ km/s; ii) the total variation in $v_{rot}(r)$ in the range 5 kpc $< r <$ 20 kpc is less than 14%;   iii) the rotational velocity $v_{rot}(LMC)$ at the LMC is in the range $(150 - 307)$ km s^{-1}.

The results obtained for these Eddington models are given in Table 3, where the first row corresponds to isotropic models. A comparison between Tables 2 and 3 shows that relaxing the requirement that the rotation curve has to be flat at distances larger than ~ 20 kpc does not change much the previous results. The main effect seems to be a larger range of variation for our results, although in Table 3 there is a trend towards a decrease of both M_H^{LMC} and N_{ev}. The values in Table 3 also show the effect of the anisotropy which increases the event duration and decreases the $[\gamma(0)/\gamma(1)]$ ratio. Notice that in both Table 2 and 3 the "minimum disk model" leads to the higher value for the dark mass (columns 4 and 5), whereas the "maximum disk model" correlates with the smaller amount in the mentioned range around the mean quantity.

Finally, we consider King-Michie models, whose results are reported in Table 4. Here, again we adopt the same strategy as in Table 2, considering only models which lead to a flat rotation curve up to the LMC distance (performing again a $\chi^2_{LMC} < 1$ test).

As before the numerical results depend on the assumed amount of luminous matter, so we consider, as an illustration, the two extreme models corresponding to the "minimum" (rows 1 and 4) and the "maximum" (rows 3 and 6) disk together with an intermediate model (rows 2 and 5).

The values in Table 4 for the King-Michie models confirm the result that the anisotropy in phase space decreases N_{ev} and $\bar{M}$, while it increases T. However, the variation is not so marked as for the Eddington models in Table 2. Moreover, contrary to Table 2, there is no correlation between the values of M_H^{LMC} and N_{ev}. This can be understood by noting that increasing the amount of luminous matter (from rows 1 to 3 for the isotropic models, and from rows 4 to 6 for the anisotropic ones) the dark matter central density decreases while the core radius a increases. Therefore, the dark matter density is higher in the central region of the galaxy as well as in the outermost regions, whereas it decreases in the space in between the Solar system and the LMC. As a result of this the amount of microlensing events N_{ev} will be smaller.

The results in Table 4 show also an increase of the total dark matter with increasing luminous matter (dominated by the disk). The increase of dark matter with a corresponding increase of luminous matter can be understood by noting that the halo density as well as the velocity dispersion of the dark matter (randomly in orbit about the common center of mass) must be sufficiently high to avoid collapse into the disk. In the following paragraph we give a qualitative argument to show that this result is physically reasonable.

Consider, first, a disk with a few dark particles distributed in a sphere. This may be thought of as an inhomogeneous sphere which must finally collapse to the disk over a sufficiently long period (due to gravitational instability). What will happen is that as the particle in the "sphere" orbits around the centre of

mass it will repeatedly cross the disk. While it is sufficiently close to the disk the particle will be attracted towards it and hence the orbit will be pulled into the plane. Now consider the sphere with only a very slight excess density in the equatorial disk. The inhomogeneity will still cause collapse into the disk but in a very much longer period. Clearly, as the density ratio of halo to excess matter tends to unity the sphere will be stabilized, whereas if it tends to zero it will be progressively de-stabilized. In other words, the time for collapse is a function of the density ratio. It tends to infinity as $\rho_H/\rho_{disk} \to 1$ and it tends to zero as $\rho_H/\rho_{disk} \to 0$.

Now we turn our attention to the results in Table 4. The larger scatter of the values in the Table compared to the ones in Table 2 can be understood as being due to the variation of the amount of luminous matter as well as to the fact that the King-Michie models have an additional parameter (the central energy cutoff parameter W_0) with respect to the Eddington models (for which $W_0 \to \infty$). Due to the presence of this additional parameter it is not possible to see a clear correlation between the microlensing results and the degree of anisotropy in velocity space, since the latter effect can be smaller than the one induced by the variation of W_0. The effect of anisotropy would show up clearly once we compare models which all have a given value of W_0.

The difficulty of getting precise microlensing results for the King-Michie models becomes even more evident if we adopt the same strategy to select physical models, which lead to Table 3, that is by considering the conditions i) - iii). In Figure 1a we plot N_{ev} as a function of the flatness degree $[v_{rot}(LMC)/v_0]$ of the galactic rotation curve. In Figure 1b, the same plot is given for the event duration T. In both figures the external and internal ellipses define the regions in which the models satisfy the condition of having a flat rotation curve only up to 20 kpc and to the LMC distance, respectively. It is clear that the knowledge of the LMC rotation velocity is crucial for determining the number of microlensing events, besides the ambiguity due to the uncertainty of the amount of the galactic luminous matter.

5. CONCLUSIONS

Several authors have studied the problem of determining the number of the expected microlensing events or, equivalently, the optical depth to microlensing by considering different models for the mass distribution, both luminous and dark in the galaxy (see, e.g. Alcock et al. 1995b, Kerins 1995, Kan-ya 1995, Evans & Jijina 1994, Evans 1994). Alcock et al. (1995b) considered "power-law models" for the halo and found that the microlensing rate can vary by as much as a factor 10 with respect to the value one gets for the standard halo. Kan-ya et al. (1995) analysed axisymmetric "power-law model" and studied the variation of the optical depth to microlensing and find that it can vary within a factor 2.5 compared to the standard spherical halo model. Similar conclusions are reached by all other authors.

The main point in our present work is that we analyse another class of models (Eddington and King-Michie) which have anisotropy in the velocity space and we determine the corresponding parameters in a self-consistent way as described in section 2. We find that with the present knowledge of the various parameters the variation in the expected number of microlensing events is at least within 30% from the value one gets for the standard halo model (flat rotation curve up to the LMC). This factor can even increase if one allows for less restrictive conditions. The typical time duration on the contrary seems to vary less, as well as the ratio $[\gamma(0)/\gamma(1)]$ which is related to the average MACHO mass. The determination of the latter with the moment method seems thus to be quite robust, in fact the ratio $[\gamma(0)/\gamma(1)]$, as compared to the value for the standard halo model, varies by at most $\sim \pm 30\%$. The main source of uncertainty, in order to be able to discriminate between different dark matter distributions and to estimate the fraction of it in form of MACHOs is due to the poor knowledge of the rotation velocity at large galactocentric distances up to the LMC.

MACHO mass in units of $M_\odot$	Mean R_E in km	Mean microlensing time T	N_{ev}
10^{-1}	3×10^8	23.5 days	5.6
10^{-2}	10^8	7.4 days	17.8
10^{-4}	10^7	17.8 hours	178
10^{-6}	10^6	1.8 hours	1784

a (kpc)	$\rho_0/10^{-3}$ ($M_\odot$ pc^{-3})	r_a (kpc)	M_H^{LMC} ($10^{11}\ M_\odot$)	M_H^{Tot} ($10^{11}\ M_\odot$)	N_{ev}	T (days)	$[\gamma(0)/\gamma(1)]$
6.3 ± 1.4	7.6 ± 1.5	∞	4.9 ± 1.0	26 ± 5	5.4 ± 0.9	22.4 ± 0.7	7.3 ± 0.2
6.5 ± 1.4	8.3 ± 1.3	47 ± 12	3.7 ± 0.8	7 ± 2	4.1 ± 0.6	27.0 ± 1.1	5.2 ± 0.3

a (kpc)	$\rho_0/10^{-3}$ ($M_\odot$ pc^{-3})	r_a (kpc)	M_H^{LMC} ($10^{11}\ M_\odot$)	M_H^{Tot} ($10^{11}\ M_\odot$)	N_{ev}	T (days)	$[\gamma(0)/\gamma(1)]$
6.2 ± 1.4	7.0 ± 3.8	∞	4.5 ± 1.5	24 ± 9	4.7 ± 1.5	22.5 ± 0.9	7.7 ± 0.2
6.4 ± 1.4	7.6 ± 4.0	40 ± 15	2.9 ± 1.5	5 ± 2	3.5 ± 1.3	27.7 ± 1.5	5.7 ± 0.3

a (kpc)	$\rho_0/10^{-3}$ ($M_\odot$ pc^{-3})	r_a (kpc)	M_H^{LMC} (10^{11} $M_\odot$)	M_H^{Tot} (10^{11} $M_\odot$)	N_{ev}	T (days)	$[\gamma(0)/\gamma(1)]$
4.9 ± 0.9	9.1 ± 0.9	∞	6.0 ± 0.9	24 ± 9	6.4 ± 0.9	22.6 ± 1.0	8.2 ± 0.8
6.6 ± 1.9	6.4 ± 0.7	∞	6.3 ± 0.9	32 ± 9	5.9 ± 0.9	21.7 ± 1.0	8.5 ± 1.0
13 ± 4	4.0 ± 0.5	∞	6.9 ± 1.0	49 ± 9	5.8 ± 0.9	19.2 ± 1.0	10.2 ± 1.1
5.7 ± 1.5	9.7 ± 1.1	40 ± 15	4.9 ± 0.7	11 ± 3	5.7 ± 0.8	25.1 ± 0.9	6.9 ± 0.6
8.9 ± 3.5	7.0 ± 1.0	42 ± 16	5.1 ± 0.8	14 ± 3	5.2 ± 0.8	24.1 ± 1.3	7.3 ± 1.1
17 ± 3	4.5 ± 0.6	45 ± 13	5.8 ± 0.7	21 ± 3	5.0 ± 0.9	21.6 ± 1.0	8.6 ± 0.8

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FIGURE CAPTIONS

Figure 1a: The number of expected microlensing events N_{ev} is given as a function of the ratio $[v_{rot}(LMC)/v_0]$ for the King-Michie models. We consider an experiment monitoring 10^6 stars in the LMC during an observation time of 1 year and we assume, as an indication, a MACHO mass of 10^{-1} $M_\odot$. Models with a flat rotation curve up to the LMC lie inside the inner ellipse (see also Table 4), whereas the ones with flat rotation curve only up to 20 kpc are bounded by the outer ellipse.

Figure 1b: The same as in Figure 1a for the average event duration T (in days).

TABLE CAPTIONS

Table 1: Galactic standard halo model.

Table 2: Mean values of parameters for Eddington models with flat rotation curves up to the LMC (the different parameters are described in the text).

Table 3: Mean values of parameters for Eddington models with flat rotation curves up to 20 kpc.

Table 4: Mean values of parameters for King-Michie models with flat rotation curve up to the LMC.

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